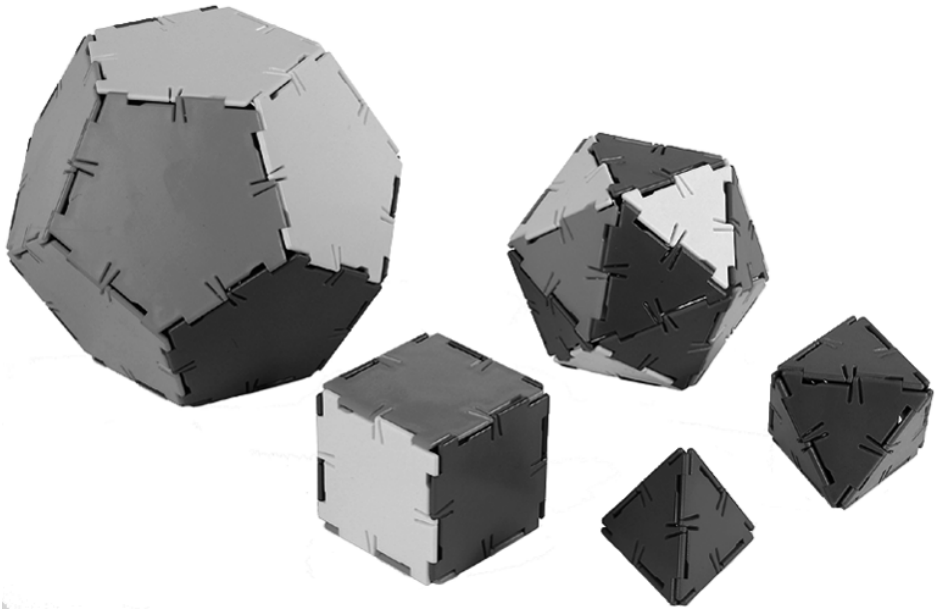
Exploring POLYDRON

Explore • Create • Understand



Bob Ansell

Contents

Introduction

This booklet aims to help teachers to understand how Polydron can improve their own teaching by providing a wide range of stimulating learning experiences for their pupils.

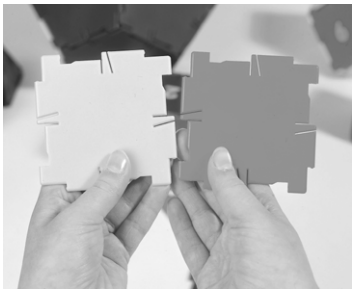
First, the guide will illustrate the range of pieces available. Then it will offer some thoughts on the use of Polydron in the mathematics classroom. This is followed by a large section given over to activities for children. There are activities for everyone from children in early years to those confident with the mathematics at the top of secondary school.

Some of the activities have been produced in a form which allows them to be enlarged on a photocopier. Others are integrated into the text as additional ideas. Either way, the activities are designed to offer a flavour of the range of mathematical work which can can covered successfully with Polydron.

Towards the back of the booklet there are some ideas for using Polydron in Technology together with details of other resource packs, including one for Number.

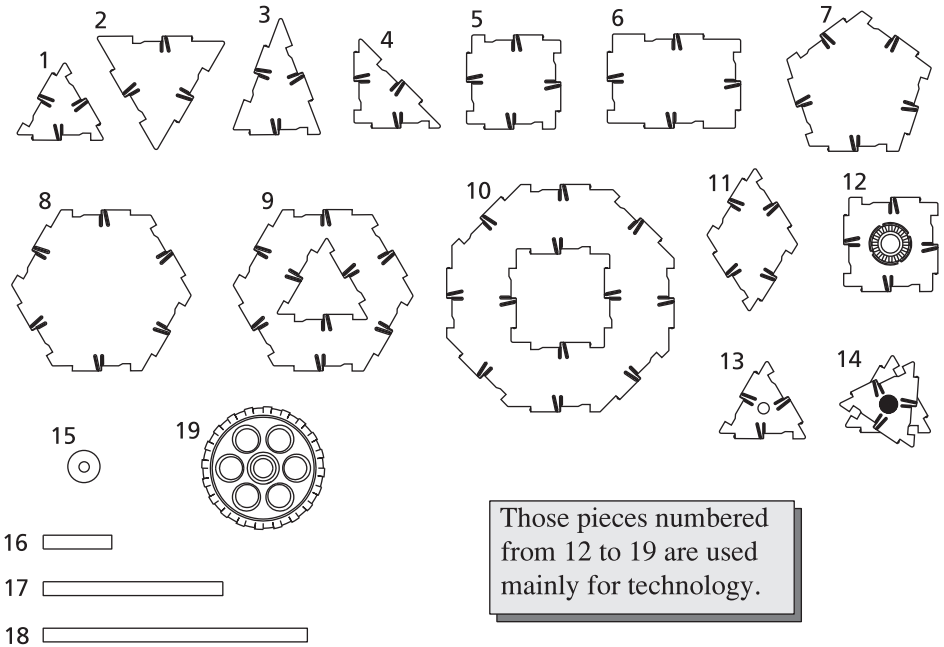
What is Polydron?

Polydron is a system of strong, colourful, red, blue, green and yellow shapes which can be fixed together, edge to edge, by means of a unique clip hinge.



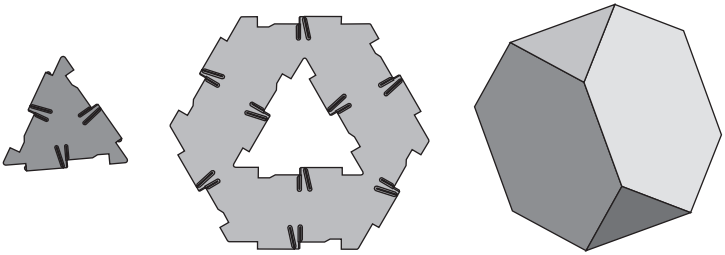
The material is designed in the United Kingdom and manufactured in China, for use in both mathematics and technology - the full list of pieces is shown below.

1	Equilateral triangle	8	Hexagon	15	Hub cap
2	Equilateral triangle (root2)	9	Hexagon with cut-out	16	67 mm strut
3	Isosceles triangle	10	Octagon with cut-out	17	170 mm strut
4	Right-angled triangle	11	Rhombus	18	250 mm strut
5	Square	12	Square with gimbal	19	Wheel
6	Rectangle	13	Triangle with peg		
7	Pentagon	14	Swivel Joint		



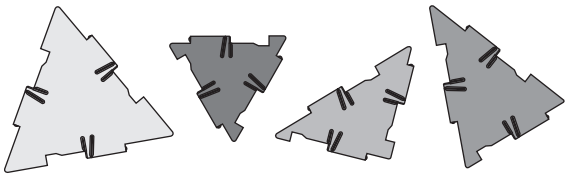
Those pieces numbered from 12 to 19 are used mainly for technology.

Some of the Polydron pieces shown opposite are designed for regular use in both mathematics and technology. For example, take four equilateral triangles and four hexagons and make the solid shown on the right.



Alternatively, the hole in the hexagon can be filled with a triangle with a peg. This is used to support struts.

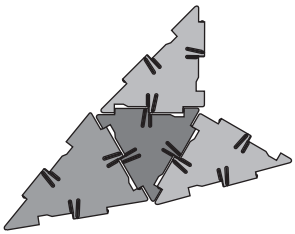
If a square is cut along a diagonal the resulting right angled triangle will have edges of two different lengths. All Polydron shapes have one of these two different edge lengths. In the diagram below you can see how these two different edge lengths are used to create four different triangles.



Notice that no other triangles are possible using just these two different edge lengths.

Take a number of each of these triangles and see how many different tetrahedra you can make.
A tetrahedron is a solid made from four triangles. Providing they fit together, you can use any sort of triangle.

Here is the net of an unusual one to get you started.



Polydron in the classroom

> The words: **Explore • Create • Understand**, as seen on the front cover are a fitting maxim for any class of young mathematicians engaged on such activities.
>
> Edith Biggs
> former HM Staff Inspector in her foreword to Primary Mathematics with Polydron

Polydron is designed to promote exploration and creativity as a vehicle for understanding through the practical application of ideas. In the classroom you can exploit the opportunities offered with Polydron material by encouraging your pupils to investigate many aspects of mathematics or technology.

Through extensive 'hands on' experience in a structured classroom environment children are better able to understand the underlying principles involved. It is also important that children should be provided with a large range of challenging work and be offered the chance to discuss the principles and practice of mathematics or technology with other children and with teachers.

• Creating the right environment •

Creating the right learning environment is not easy. Polydron is well known for its use as a motivator in free play and it has great strength in this context. However, the benefits of careful planning by teachers and the use of reflective intervention to support and extend children's learning turn a good resource into a great one.

Many teachers would agree that they are most successful when they are able to focus the attention of children on a particular aspect of mathematics or technology by using a stimulus. Polydron provides not only that stimulus to learning but also a focus for discussion, a means of recording an activity by making something and, perhaps most importantly, a way for pupils to gain satisfaction by completing a challenge. When using Polydron always try to see beyond the immediate construction.

Above all, try to recognise and exploit the opportunities for intellectual curiosity and challenge.

Polydron in Mathematics

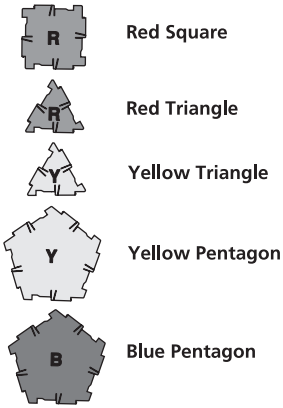
Polydron has a place in mathematics for all pupils whatever their age. On the next few pages some activities will illustrate the range and versatility of the material. Many of the activities are adapted from Polydron resource publications. More details of these can be found on pages 21 and 22.

The activities have been organised to suit a number of purposes. For example, they will show the possibilities offered by Polydron in both technology and mathematics in a form which can be used to introduce the ideas to a group of children. Alternatively, some of the pages may be enlarged on a photocopier and used as worksheets to support your own scheme of work.

• Early Years •

Sorting is a natural activity with Polydron. Pieces can be sorted with simple attributes, such as shape and colour and also more difficult ones such as regularity. In this way Polydron provides a useful alternative to logiblocs or other sorting materials.

Sorting activities can be extended in many ways. The activity described below is called **one difference dominoes** and is ideal for pairs or small groups.



The first player places a piece, such as a red square, on the table.

The next player in turn then places a piece next to it. The piece chosen must differ in one attribute from the previous piece. In the sequence shown you can see how one attribute is altered at each step.

To encourage mathematical language, try to insist that the change in attribute is spoken as the piece is played. For example, when moving down the sequence from a red square to a red triangle a player might say, “Same colour - different shape”.

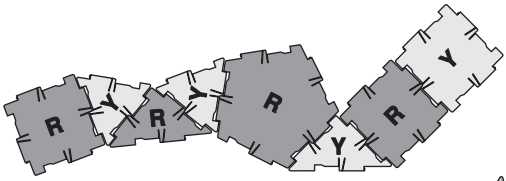
If a player cannot go then that turn is missed. The winner of the game is the last player to place a piece.

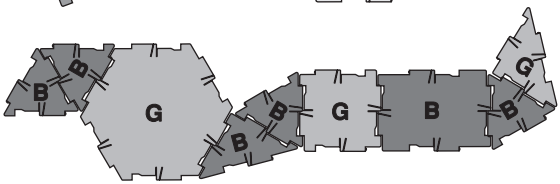
This activity and those on pages 8 and 9 are adapted from Primary Mathematics with Polydron. They illustrate how Polydron can be used to assist your work with pattern, 3D shape and 2D shape.

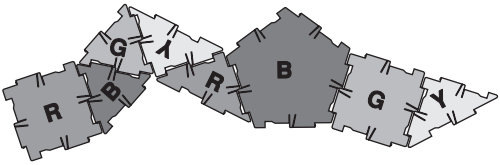
Continue these patterns

Polydron Activity 1 Sheet

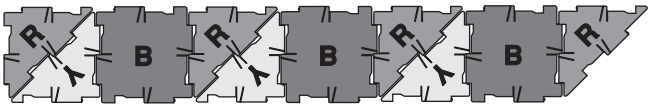
❏ Continue the colours.

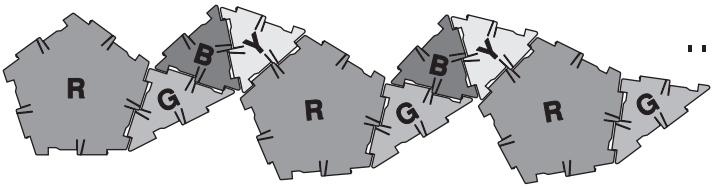
1  .....

2  .....

3  .....

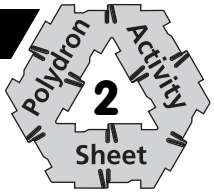
❏ Continue the colours and the shapes.

4  .....

5  .....

❏ Now make some patterns of your own.

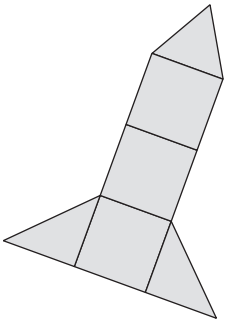
Make these shapes

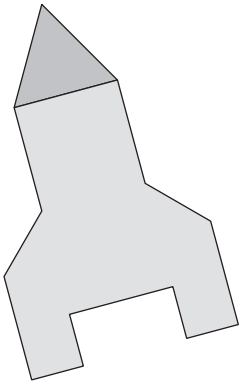


❏ Make this flat rocket shape.

❏ Can you make it longer?

❏ Can you make it wider?



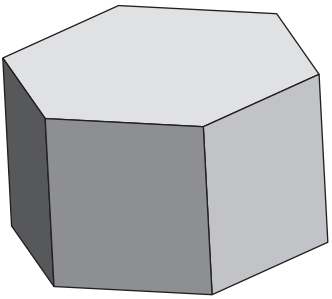


You can only see the outline of this rocket.

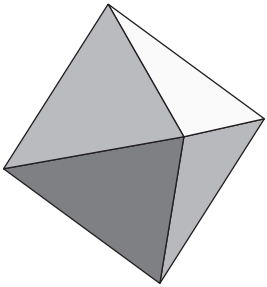
❏ Can you make it?

❏ Make these two solids. Use the pieces shown..

2 hexagons and 6 squares

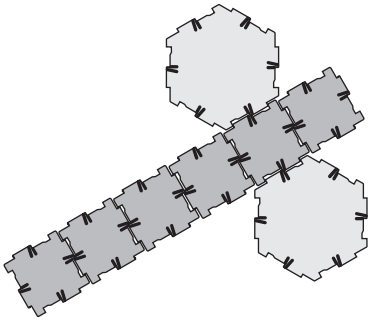


8 triangles



• Mathematics in the Middle Years •

Children need a wide variety of shape and space activities during their middle years. They need to extend and develop their understanding of the properties of 2D shapes, the families of 3D shapes and their nets. Many of the best examples begin with a teacher-led activity such as that shown here.



What solid will this net fold into?

Alternatively, provide a solid and see if children can construct the net without unfolding it first.

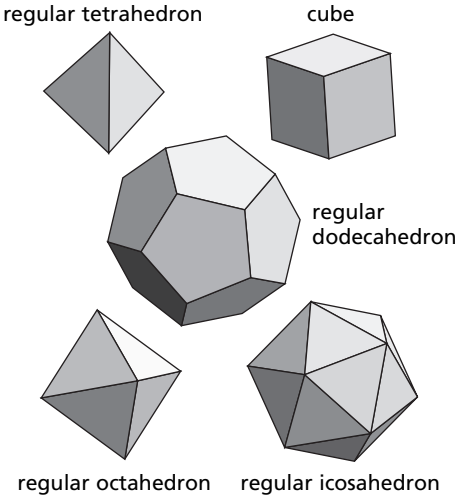
Children should try to describe what the net will look like. They can make one and then test their solutions by unfolding the solid.

A great strength of Polydron has always been the opportunities it offers to make, describe and learn about 3D solids. Perhaps the most important family of solids which children encounter is the family of Platonic solids.

This family of five solids underpins much of our solid geometry and our architecture. Each solid is made from just one sort of polygon and at every corner the same number of pieces meet.

For those who want to delve more deeply into this family, there is a Polydron publication devoted to it. See page 22 for more details.

The Platonic Solids



While it is important during the middle years that children come to focus upon the properties of geometric shapes, to make generalisations about these properties and to classify shapes, it is also important to develop children's ability to visualise and work with geometric images.

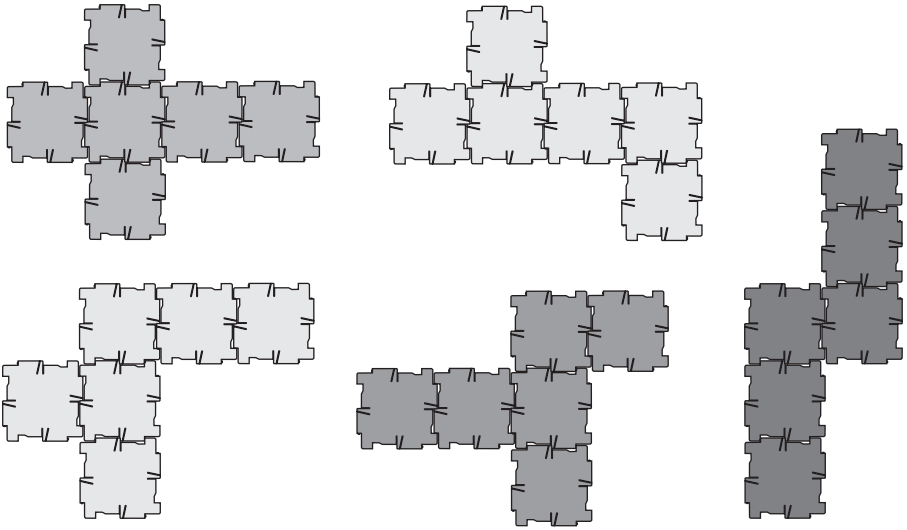
The activities on the next few pages are adapted from Primary Mathematics with Polydron and offer some ideas with each of these aspects of mathematics.

Nets of cubes and pyramids

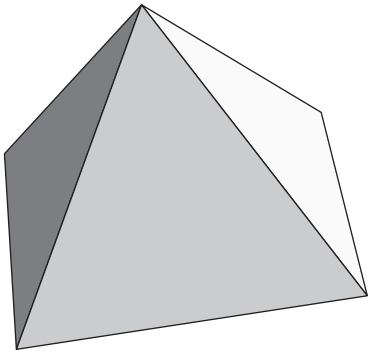
Polydron Activity 3 Sheet

❏ Which of these nets will fold to make a cube?

❏ Make each one and check.



❏ Find other nets of a cube and draw them on squared paper.



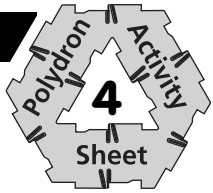
Here is a square-based pyramid.

❏ Make it and then unfold it to form a net.

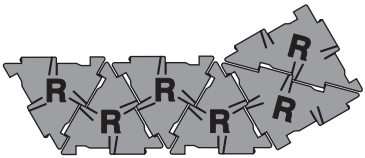
❏ Draw your net.

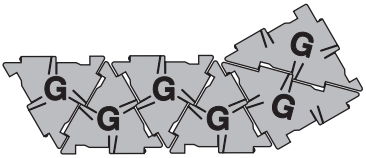
❏ Find as many different nets of this pyramid as you can. Draw each one.

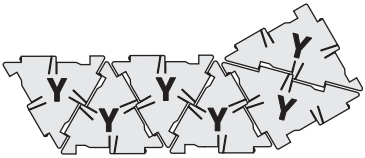
A four piece puzzle

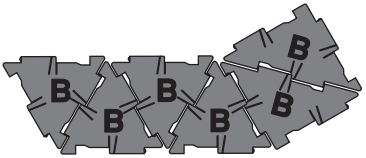


❏ Make these four nets. Each one needs 4 equilateral triangles and 2 right-angled triangles.





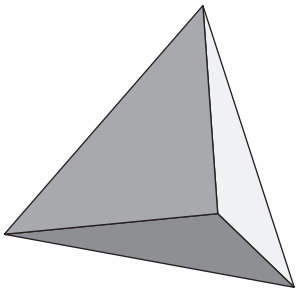


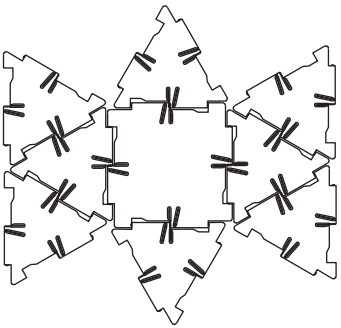


❏ Fold and join up each of these nets to make four identical closed solid shapes.

❏ Place all four solids together to make up a large tetrahedron.

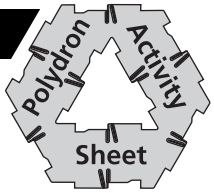
❏ Give the puzzle to someone else to try.



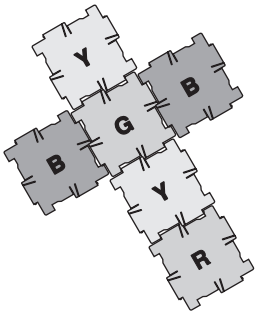


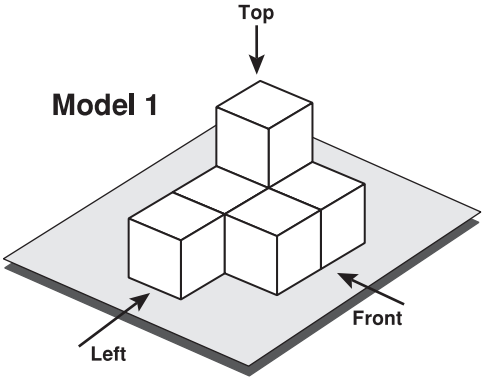
❏ Make a different version of the puzzle from two copies of this net.

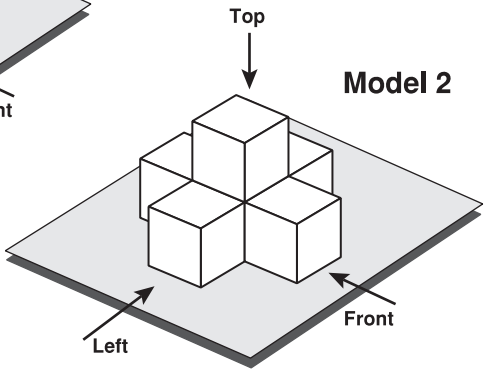
Plans and elevations

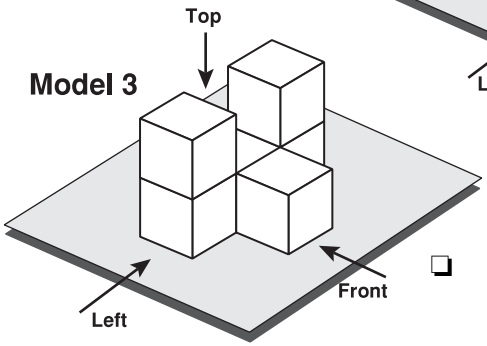


- ❏ Make six Polydron cubes using the net shown here and place them all together on a table with the red faces up.
- ❏ Make each of the models below. Twist the cubes to get one colour in each view.
- ❏ For each model, use square dotted paper to draw the views from above, from the left and from the front.







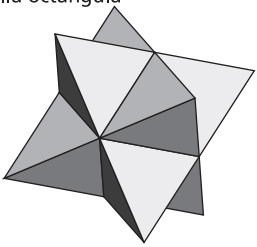


- ❏ Make some models of your own and draw the views you get from above, from the left and from the front.

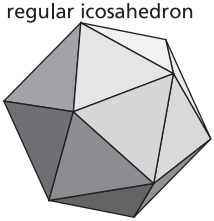
Some solids to make

Polydron Activity 6 Sheet

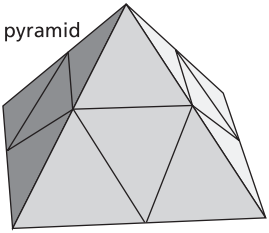
stella octangula



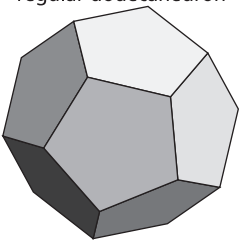
regular icosahedron



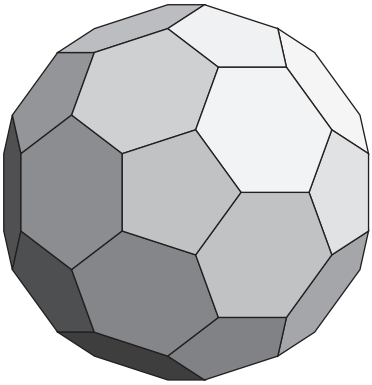
pyramid



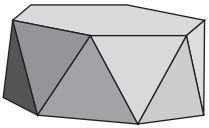
regular dodecahedron



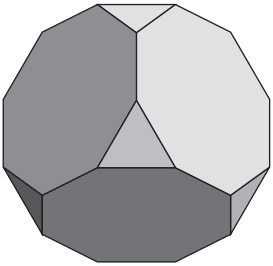
truncated icosahedron



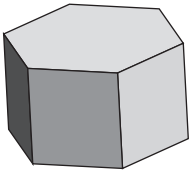
hexagonal antiprism



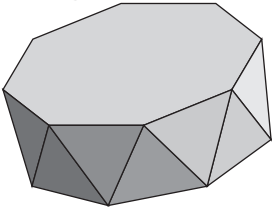
truncated cube



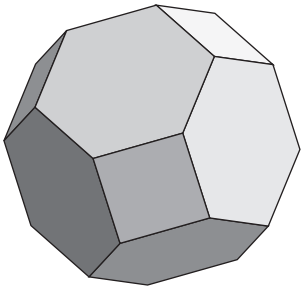
hexagonal prism



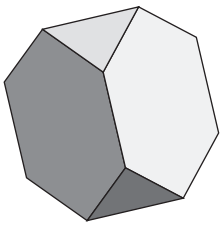
octagonal antiprism



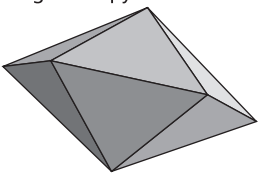
truncated octahedron



truncated tetrahedron



pentagonal dipyramid



• Mathematics in the Secondary School •

For many children, moving from primary to secondary school means they leave behind them many of the aids to understanding. Polydron is often neglected in the secondary years in favour of an 'abstract approach' to geometry or shape and space.

This is a pity, since the material offers unique opportunities to link the concrete and the abstract, and to provide a real framework upon which many of the abstractions of geometry depend.

For example, when exploring Euler's relation between the number of faces, vertices and edges of a solid, it is much better to construct your own solids to gain some insight as to why the formula works. If you are not familiar with the formula then make some solids and fill in the entries in the table below. Try to find a simple relation between the number of faces, vertices and edges of your solids.

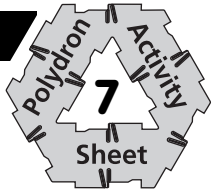
Name of Solid	Number of faces (F)	Number of vertices (V)	Number of edges (E)
Tetrahedron	4	4	6
Cube	6	8	12

In addition to work with shape and space, Polydron provides a rich medium for working with algebra, particularly when developing the all important sense of algebra as generalisation of arithmetic. This aspect of algebra is often underplayed, while the abstract side is promoted. Polydron is an ideal medium to provide a practical context to promote generalisation. In addition, Polydron provides a focus for mathematical language and the development of proof and rigour.

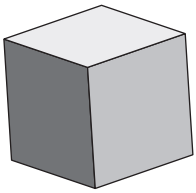
The activities on the next few pages are adapted from Mathematics with Polydron and illustrate the 3D strengths of Polydron and the use of the material to promote mathematical language. Also, the activity called ***cutting corners*** illustrates how a practical approach can stretch even the most able.

Finally, the page of solids shown opposite illustrates how this versatile material can be used to create many solids which exhibit their own mathematical beauty. This simple activity is suitable for a wide range of ages and attainment.

Logi-Solids



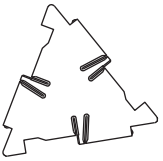
❑ Build each of these solids by using the clues marked with a ◆ shape.



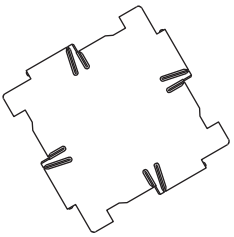
1 This solid is a cube.

- ◆ Only three colours are needed.
- ◆ One blue square touches all three red squares.
- ◆ Another blue square touches only two red squares.
- ◆ A yellow square touches only one blue square.

2 This solid needs eight equilateral triangles.

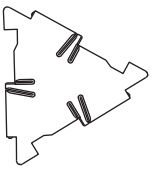


- ◆ Yellow triangles are always joined to red triangles.
- ◆ There are four yellow triangles.
- ◆ Only two colours are used.



3 This solid is made only from squares and equilateral triangles.

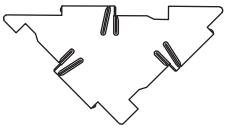
- ◆ A red square has a yellow triangle joined to each edge.
- ◆ Green triangles touch only yellow shapes.
- ◆ There is one yellow square in the solid.
- ◆ There are no blue squares.
- ◆ There are ten pieces altogether.

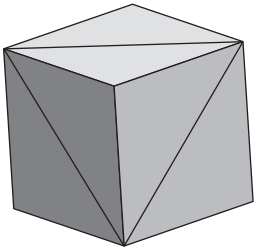


Cutting corners

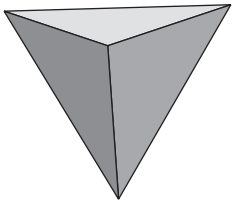
Polydron Activity 8 Sheet

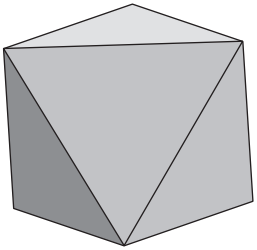
❏ Construct a cube using 12 right-angled triangles. Make one corner of your cube the same as the one in the picture.





❏ Remove this corner, as shown and cover the hole with a large equilateral triangle.



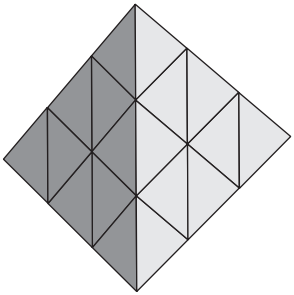


❏ What fraction of the volume of the cube have you removed?

❏ Find a way to cut four corners off this large tetrahedron.

What shape have you left?

What fraction of the volume have you removed?



Technology with Polydron

A lot of technology work in primary schools tends to focus on design skills at the expense of working with materials and making things. With Polydron you have an an ideal resource to focus upon making and testing models or structures.

The activity illustrated opposite is adapted from Technology with Polydron, a resource pack which is designed to foster the skills of planning, evaluating and testing throughout the construction process.

Technology with Polydron can provide children with specific opportunities to:

- select appropriate materials to suit a given purpose
- assemble, join and combine a range of components using a variety of techniques
- work through a series of stages in the making of a product, including the use of prototypes
- test and evaluate a product against design criteria, identify strengths and weaknesses and implement improvements
- become competent at matching their methods of working to the materials and the task in hand including the limitations of particular materials
- understand ways of working with construction kits which allow both frameworks and rigid body models to be made
- develop an understanding of the behaviour of structures, including how they can fail when loaded

The model shown on the worksheet is a suspension bridge. This provides many opportunities for children to study all aspects of the design process, including testing the strength of the bridge by adding masses until it collapses. Further, the analysis of this failure provides a basis to improve the design.

The resource pack itself offers an extensive range of activities which cover many aspects of technology in the primary school and give children the chance to create simple fairground models, towers and a multi-storey car park.

A suspension bridge

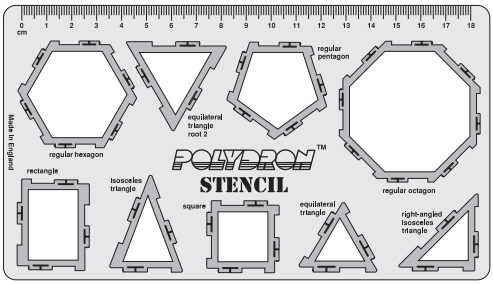
You will need
4 octagons, 8 rectangles with cut-outs and 13 other rectangles
8 squares with gimbals, 4 squares with holes and 18 other squares
4 medium struts, 8 small equilateral triangles
A ball of thin string for the suspension wires and supports strings.

- ❏ Look carefully at the picture.
- ❏ Use the octagons to make a base for each end of the bridge.
- ❏ Build each tower, making sure you put the squares with gimbals in place with a strut which passes through the tower.
- ❏ Cut two lengths of thin string to be the main suspension wires. Make each length of string about 2 metres. You may not need all of the string now but it will be useful later.
- ❏ Now cut and fix small pieces of string for the vertical supports, wedging the ends into the rectangles. Wind each end of the main support wires around the struts and then secure them in the groove of a Polydron piece.

• The Polydron Stencil •

The stencil allows children to draw nets, tessellations and other figures where Polydron or Frameworks shapes are joined together.

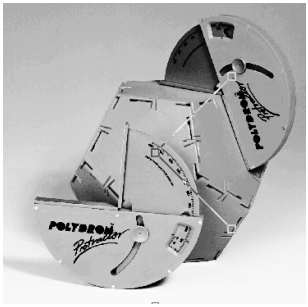
The shorter edge length in the polygons shown in the stencil is 20mm. The longer edge is 20mm x √2, or a little over 28mm.



• The Protractor •

This protractor is specially designed to help pupils with angle measure when working with Polydron.

It is ideal for measuring the angles of a solid figure such as the one in the photograph. In this case a conventional protractor is unsuitable.



Other Polydron and Framework Resources

Mathematics with Polydron

This resource book is aimed at older pupils. It offers teachers the same style and support as the primary book and provides 36 worksheets covering not only 3D and 2D shape and space but also algebra. The book provides open-ended material and problem-solving activities throughout.

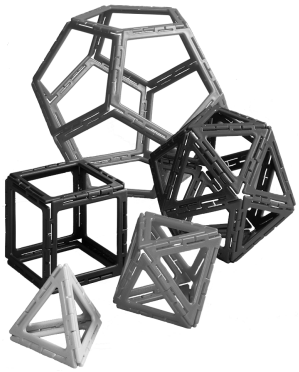
Polydron Geometry Set

This is a set of Polydron shapes which has been carefully put together to provide sufficient material to carry out all of the activities in Mathematics with Polydron.

Platonics with Frameworks

This pack provides an in-depth look at an important family of solids. It covers a great deal of mathematics, including proof and the use of specialist notation.

The pack also offers a historical framework to the Platonic solids.



Acknowledgements

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